



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

EARTHQUAKE RESISTANT DESIGN OF FOUNDATIONS:

Lecture 11: Footings under Dynamic Loads: Analytical Approaches

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ERD of Foundations - Modules

1. Introduction
- 2. Shallow Foundations**
3. Pile Foundations
4. Well Foundations
5. SSI for Deep Foundations



Module - 2

Shallow Foundations



Module -2: Chapters

1. General Requirements
2. Bearing Capacity and Settlement
3. Combined Footings
4. Raft Foundations
5. Footings Subjected to Dynamic Loads

Chapter 5: Footings Subjected to Dynamic Loads

- **Dynamic Loads and Methods of Analysis**
- **Pseudo-Static Analysis**
- **Earthquake Load: Horizontal Load and Moment**
- **Eccentric Load on Footings**
- **A new Method for Design of Eccentrically Loaded Footings**
(a) Bearing Capacity (b) Settlement
- **New Method: (a) Design Procedure (b) Design Example**
- **Inclined Load on a Footing**



Dynamic Loads and Methods of Analysis

- Foundation are subjected to dynamic loads due to
 - a. Earthquake (Predominantly Horizontal)
 - b. Blasting (Predominantly Vertical)
 - c. Operation of Machine (Coupled)
- Methods of Analysis
 - a. Pseudo-Static Analysis
 - b. Dynamic Analysis

Dynamic Analysis

All analytical approaches are based on the assumption that soil rupture under transient loads occurs along a static rupture surface

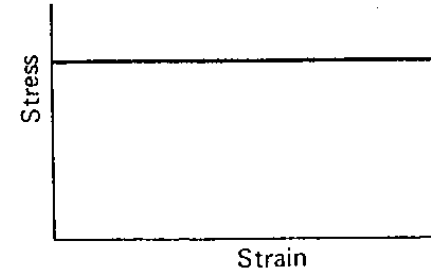
- Transient Vertical (rotation) load (Triandafilidis, 1965)
- Transient Vertical (reciprocating) load (Wallace, 1961)
- Transient Horizontal (reciprocating) Load (Chummar, 1965)
- Numerical Examples

Triandafilidis's Solution

Triandafilidis (1965) studied the responses of footings in saturated cohesive soils subjected to vertical dynamic load pulses of defined shape.

Assumptions

1. The failure surface of soil is cylindrical
2. The saturated cohesive soils behaves as a rigid plastic material
3. The forcing function is assumed to be an exponentially decaying pulse
4. The influence of strain rate on the shear strength of the soil and the dead weight of the foundation are neglected



Triandafilidis's Solution (Continued...)

The decaying stress at time t is given by

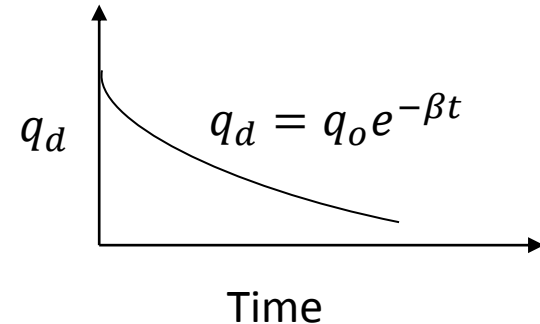
$$q_d = q_o e^{-\beta t} = \lambda q_u e^{-\beta t} \quad (1a)$$

β = decaying function

q_u = static bearing capacity of continuous footing

q_o = instantaneous peak intensity of stress pulse

$$\lambda = \text{over load factor} = \frac{q_o}{q_u} \quad (1b)$$



Triandafilidis's Solution (Continued...)

The equation of motion,

Moment of disturbing forces = Moment of resisting forces
(about O point)

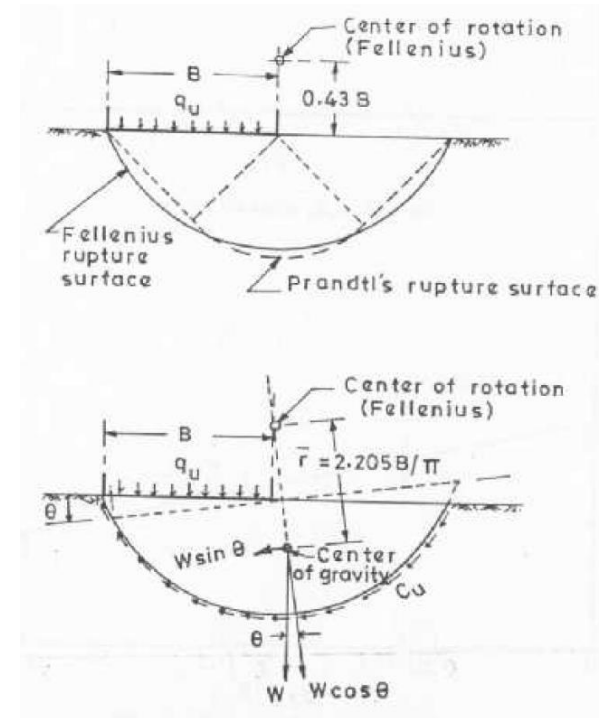
Deriving moment M_{dp} due to applied dynamic pulse is given by

$$M_{dp} = \frac{1}{2} q_d B^2 \quad (2a)$$

The static bearing capacity of a continuous footing along the failure surface

$$q_u = 5.54 c_u$$

c_u = undrained shear strength



Triandafilidis's Solution (Continued...)

The resisting moment M_{rs} due to shear strength

$$M_{rs} = \frac{1}{2} q_u B^2 \quad (2b)$$

Where B is width of footing

The resisting moment M_{ri} due to the rigid body motion of the failed soil mass

$$M_{ri} = J_o \ddot{\theta}$$

Where, B is width of footing

$$J_o = \text{polar mass moment of inertia} = \frac{W B^2}{1.36g}$$

$$W = \text{Weight of cylindrical soil mass} = 0.31\gamma\pi B^2$$

γ = Unit Weight of soil

$$M_{ri} = \frac{W B^2}{1.36g} \ddot{\theta} \quad (2c)$$

Triandafilidis's Solution (Continued...)

The displaced position of the soil mass generates a resisting moment M_{rw}

$$M_{rw} = W\bar{r} \sin \theta = W\bar{r}\theta = W \frac{2.205B}{\pi} \theta \quad (2d)$$

By equating the moment of deriving forces to the resisting forces

$$M_{dp} = M_{rs} + M_{ri} + M_{rw} \quad (3)$$

$$\frac{1}{2} q_d B^2 = \frac{1}{2} q_u B^2 + \frac{W B^2}{1.36g} \ddot{\theta} + W \frac{2.205B}{\pi} \theta$$

$$\frac{1}{2} q_d B^2 - \frac{1}{2} q_u B^2 = \frac{W B^2}{1.36g} \ddot{\theta} + W \frac{2.205B}{\pi} \theta$$

$$\frac{1}{2} q_u B^2 (\lambda e^{-\beta t} - 1) = \frac{W B^2}{1.36g} \left(\ddot{\theta} + \frac{2.205}{\pi} \frac{1.36\theta}{B} \right)$$

$$\ddot{\theta} + \frac{3g}{\pi B} \theta = \left(\frac{0.68g}{W} \right) q_u (\lambda e^{-\beta t} - 1) \quad (4)$$

Equation 4 is called equation of motion

Triandafilidis's Solution (Continued...)

The natural frequency and time period of system are given by

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3g}{\pi B}} \quad (5a)$$

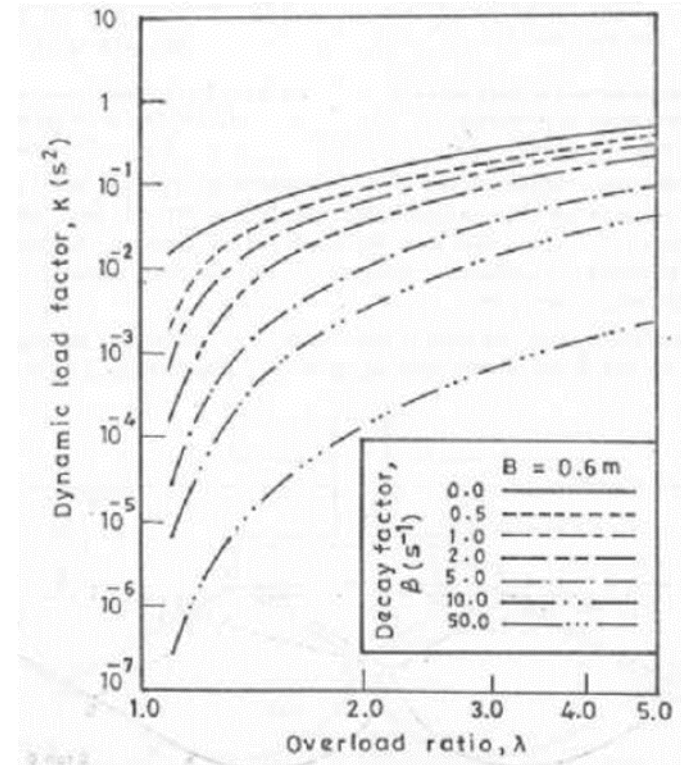
$$T = \frac{1}{f} = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{\pi B}{3g}} \quad (5b)$$

Rearranging equation 4,

$$\left(\frac{W}{0.68gq_u}\right) (\ddot{\theta} + \frac{3g}{\pi B} \theta) = (\lambda e^{-\beta t} - 1)$$

$$K = \frac{W}{0.68gq_u} \theta_{max} = \text{dynamic load factor (in } s^2)$$

The charts for K for B = 0.6, 1.5 and 3m are given



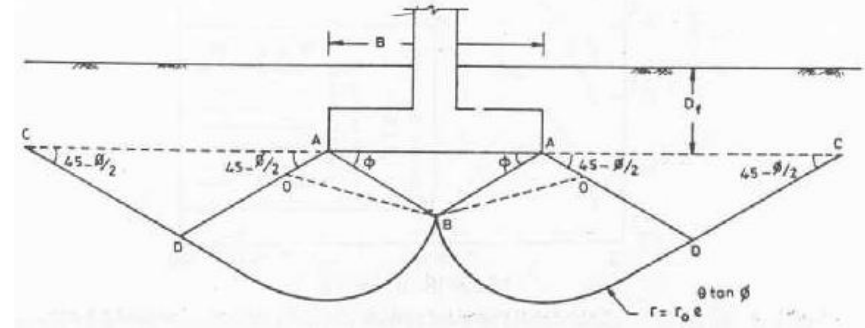
Wallace's Solution

Analysis presented by Trianadafilidis (1965) is based on rotational mode of failure. However, it is possible that a foundation may fail by vertically punching into the soil mass due to the application of vertical transient load. Wallace (1961) presented a procedure for the estimation of the vertical displacement of continuous footing considering punching mode of failure.

Wallace's Solution (Continued...)

Assumptions:

1. The failure surface in the soil mass is assumed to be similar as suggested by Terzaghi (1943) for the evaluation of static bearing capacity of strip footings.
2. The soil behaves as a rigid plastic material
3. The ultimate shear strength is given by
$$\tau = c + \sigma \tan \phi$$
4. The dynamic load applied to the footing is initially peak triangular force pulse
5. The footing is assumed to be weightless and to impart uniform load to the soil surface



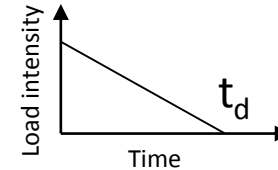
Wallace's Solution (Continued...)

Analysis

The applied load is assumed to be an initial-peak triangular force which decays to zero at time t_d . The peak load q is expressed in pressure units. Since the function is discontinuous at time t_d , two equations are necessary

For $0 \leq t \leq t_d$, Loading function = $qB(1 - \frac{t}{t_d})$

For $t \geq t_d$, Loading function = 0



In Terzaghi Failure surface , BD is an arc of a logarithmic spiral with its centre at O. It is defined by

$$r = r_o e^{\theta \tan \phi}$$

r_o = distance OB, ϕ = angle of internal friction

Wallace's Solution (Continued...)

The static bearing capacity q_u for such a failure surface is given by

$$q_u = cN_c + \gamma D_f N_q + \frac{1}{2} \gamma B N_\gamma$$

The inertial force is given by, $I_f = N_I \gamma B^2 \frac{d^2 \Delta}{dt^2}$

Δ = Displacement at any time t

N_I = Coefficient of dynamic inertial shear resistance (dimensionless)

The coefficient N_I depends on ϕ and $K = 2 \cdot OA/B$

The moment due to displacement

$$R_F = N_R (B\gamma) \Delta$$

The coefficient N_R (dimension m/s^2) also depends on ϕ and K

Wallace's Solution (Continued...)

Bearing Capacity Factors (N_γ , N_c , N_q , N_l , N_R)							
ϕ (deg)	K	N_γ	N_c	N_q	N_l	N_R	$\sqrt{\frac{N_R}{N_l}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	- 0.05	0.0000	5.7277	1.0	0.0633	2.0125	5.6366
	0.00	0.0000	5.7124	1.0	0.0631	1.9723	5.5887
	+ 0.05	0.0000	5.7258	1.0	0.0633	1.9433	5.5394
5	- 0.65	0.1454	79.6255	7.9664	0.3755	8.9076	4.8709
	- 0.60	0.1445	29.8163	3.6086	0.2280	6.4362	5.3126
	- 0.55	0.1481	18.9958	2.6619	0.1579	5.0332	5.6460
	- 0.50	0.1553	14.3469	2.2552	0.1213	4.1699	5.8636
	- 0.45	0.1655	11.8179	2.0339	0.1011	3.6088	5.9750
	- 0.40	0.1786	10.2699	1.8985	0.0897	3.2299	6.0020
	- 0.35	0.1945	9.2580	1.8100	0.0833	2.9674	5.9698
	- 0.30	0.2131	8.5723	1.7500	0.0799	2.7828	5.9005
	- 0.25	0.2344	8.1007	1.7087	0.0786	2.6523	5.8108
	- 0.20	0.2585	7.7778	1.6805	0.0785	2.5604	5.7116
	- 0.15	0.2855	7.5629	1.6617	0.0793	2.4969	5.6099
	- 0.10	0.3154	7.4291	1.6500	0.0809	2.4547	5.5096
	- 0.05	0.3483	7.3580	1.6437	0.0829	2.4288	5.4128
	0.00	0.3843	7.3366	1.6419	0.0853	2.4155	5.3205
	+ 0.05	0.4233	7.3553	1.6435	0.0881	2.4122	5.2330
	- 0.60	0.5700	53.9491	10.5127	0.1120	5.7922	7.1922

Wallace's Solution (Continued...)

The differential equations are setup by equating the four vertical forces to zero. There must be separate equations before and after time t_d .

$$\text{For } 0 \leq t \leq t_d \quad \frac{d^2 \Delta}{dt^2} + \frac{N_R}{N_I B} \Delta = \frac{q - q_u}{N_I \gamma B} - \frac{q}{N_I \gamma B t_d} t$$

$$\text{For } t \geq t_d \quad \frac{d^2 \Delta}{dt^2} + \frac{N_R}{N_I B} \Delta = -\frac{q_u}{N_I \gamma B}$$

The solution of the differential equations will yield equations of footing displacement versus time as:

$$\Delta = C_1 \cos(K' t) + C_2 \sin(K' t) + \left(\frac{q - q_u}{N_R \gamma} \right) - \left(\frac{q}{N_R \gamma t_d} \right) t$$

$$\text{and} \quad \Delta = C_3 \cos(K' t) + C_4 \sin(K' t) - \left(\frac{q_u}{N_R \gamma} \right)$$

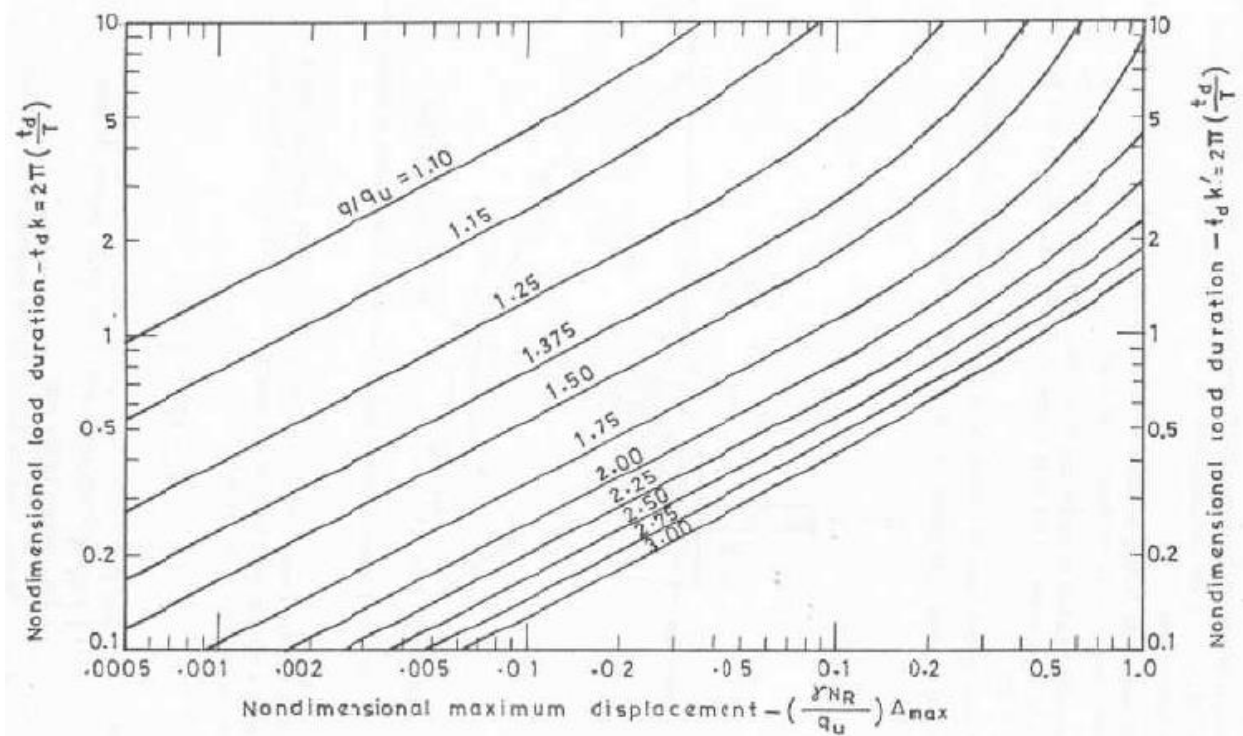
Where, $K' = \sqrt{\frac{2N_R}{N_I B}}$ and C_1, C_2, C_3 and C_4 are coefficients of integration.

Wallace's Solution (Continued...)

The C_1 and C_2 are determined by initial conditions. While C_3 and C_4 are determined by conditions at $t = t_d$

The coefficients N_γ , N_c , N_q , N_l and N_R are dependent only on values of ϕ and K'

Δ_{max} can be determined from the graph.



References

1. **Prakash S. (1981), "Soil Dynamics", McGraw-Hill Company, New York**
2. **Saran S. (2006), "Soil Dynamics & Machine Foundation", Galgotia Pub. Pvt. Ltd, New Delhi.**

Thank You

